Securitization and Risk Transfer

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Outline

- Introduction
  - The idea of securitization
  - Non-financial risk factors
  - Indifference valuation
- Equilibrium pricing in incomplete markets: the one period model.
- Equilibrium pricing in incomplete markets: the multi-period model.
- Computing equilibria in discrete time: backward stochastic difference equations.
- Backward stochastic differential equations
- Cross hedging in continuous time.
- Equilibrium pricing in continuous time.

Related Literature

- H & Müller (2007) "On the spanning property of risk bond priced by equilibrium", Math. Operations Research.
- H, Pirvu & dos Reis (2008) "On securitization, market completion and equilibrium risk transfer", Working paper.
- Cheridito, H, Kupper & Pirvu (2008) "Equilibrium in incomplete markets under translation invariant preferences", in preparation.
- Filipovic & Kupper (2007) "Equilibrium prices for monetary utility functions", Working paper.

Securitization and Risk Transfer

- Convergence of insurance and financial markets.
- Securitization: Transformation of non-tradable risk factors into tradable financial securities with the goal of transferring external risks to capital markets.
- Economic problems related to securitization:
  – cross-hedging of financial securities;
  – equilibrium pricing in incomplete markets;
  – ...
- Mathematical problems related to securitization:
  – utility optimization in incomplete markets;
  – backward stochastic differential equations;
  – ...

THIS MINI-COURSE DISCUSSES RECENT RESULTS ON PRICING, MANAGING, MITIGATING AND TRANSFERRING EXTERNAL (NON-FINANCIAL) RISKS.

Weather: A Non-Financial Risk Factor

- About 1/7-th of the US economy is weather sensitive:
  - agriculture (temperature, precipitation, ...)
  - energy sector (temperature, tropical storms, hurricanes, ...)
  - travel & recreation (temperature, precipitation, ...)
- Underlying risk factors (e.g. temperature) are non-tradable.
- Catastrophic risk: hailstorms, hurricanes, earthquakes, ...
- Non-Catastrophic risk: non-extreme deviations from the norm.

"A few degrees colder and this were winter-wonderland."
"The greenest winter olympics ever."

- Insurance losses from weather phenomena are often quite high.
- Securitization: Transform insurance risks to capital markets.

THE PRICING OF WEATHER DERIVATIVES IS OUTSIDE THE REALM OF TRADITIONAL MATHEMATICAL FINANCE.

Beyond Financial Valuation Principles

- Standard valuation schemes for stock-derivatives such as

$$\max\{S_T - K, 0\}$$

  use replication arguments; preferences are not important.

- This requires the underlying to be tradable (and storable.
- It also requires the market to be sufficiently liquid.

- These assumptions are not satisfied for weather derivatives:
  - temperature and precipitation are non-tradable underlyings;
  - the market for weather related securities is rather illiquid.
  - the market for weather related securities is incomplete.

- Replication arguments do not apply to weather derivatives.

OUR FOCUS IS ON PREFERENCE-BASED VALUATION SCHEMES.

Measuring Risk

- Mitigating risk requires a method of measuring risk.
- A risk measure is a map $\varrho : \mathscr{X} \to \mathbb{R}$ from the set of financial positions (random variables) $\mathscr{X}$ such that for $X, Y \in \mathscr{X}$:

$$\varrho(X) \leq \varrho(Y) \quad \text{if} \quad X(\omega) \geq Y(\omega)$$

and

$$\varrho(X + m) = \varrho(X) - m \quad \text{for all constants } m \in \mathbb{R}.$$

- The latter property is called "cash invariance;" interpret $\varrho(X)$ as a capital requirement.
- A desirable property is convexity (diversification):

$$\varrho\left(\lambda X + (1 - \lambda) Y\right) \leq \lambda \varrho(X) + (1 - \lambda)\varrho(Y).$$

We assume that agents' risk preferences are translation invariant.

Indifference Valuation

- One approach to securitization is utility indifference valuation:

$$\inf_{F\in\mathscr{X},\pi} \varrho_A(\xi_A - F + \pi) \quad \text{subsect to} \quad \varrho_B(\xi_B + F - \pi) \leq \varrho_B(\xi_B).$$

- This yields an optimal claim F^* and a (benchmark) value π^*.
- Indifference valuation assumes a high degree of asymmetry.
- It is more about risk sharing (reinsurance) then risk transfer.
- Typically, π^* is a benchmark rather than a transaction price ...
  ... unless there is a large set of homogeneous buyers
  ... with no market power (impact on transaction prices).
- Few buyers: is indifference valuation still appropriate?

We move away from the idea of indifference valuation and consider models of *equilibrium pricing*.